Direct Integration of Differential Equations and Curve Fitting Project
Consider the following mathematical relationship

\[ P = k \delta \]  

(1)

where

\[ k = c L^b \]  

(2)

so

\[ P = (c L^b) \delta \]  

(3)

The variables \( c \) and \( b \) are constant real numbers that you will solve for later in the assignment.

You are given some experimental data in Figure 1, which is summarized in Table 1. You are asked to determine the relationship between the variable, \( k \), and the variable, \( L \).

\[ \text{Table 1. Summary of } k \text{ for various values of } L. \]

<table>
<thead>
<tr>
<th>( L )</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>111.20</td>
</tr>
<tr>
<td>15</td>
<td>54.67</td>
</tr>
<tr>
<td>18</td>
<td>33.90</td>
</tr>
<tr>
<td>21</td>
<td>22.50</td>
</tr>
<tr>
<td>26</td>
<td>11.36</td>
</tr>
<tr>
<td>30</td>
<td>7.15</td>
</tr>
</tbody>
</table>

Figure 1. Experimental Data collected for \( P \) vs. \( \delta \) for various values of \( L \).

Equation 3 is based on the following governing differential equation

\[ \frac{d^2y}{dx^2} = \frac{P(L-x)}{EI} \]  

(4)
where $E$, $I$, and $L$ are all constants. The value for $\delta$ is the same as $y(x)|_{x=L}$. The boundary conditions for this second-order differential equation are

$$y(0) = 0$$
$$\frac{dy}{dx} \bigg|_{x=0} = 0$$  \hspace{1cm} (5)

In this project you will determine the theoretical solution by solving the governing differential equation for this system and compare this theoretical result to the experimental results given in Table 1. Finally, you will use the experimental results (given in Table 1) along with the equation for $I$ (given below) and the theoretical solution to estimate the value of the constant, $E$.

1. Solve equation (4) subject to the boundary conditions given in equations (5) and use your result to determine the theoretical relationship between the $\delta$ and the parameters in equation (2).

2. The constant $I$ is sometimes called the second moment of area and is determined by the following equation:

$$I = \int_A y^2 dA$$  \hspace{1cm} (6)

where $A$ is an area of integration and $y$ is measured from the x-axis that goes through the center of the area. Show that for a rectangular area with a width, $w$, and a thickness, $t$, the moment of inertia is

$$I = \int_A y^2 dA = \frac{1}{12} wt^3$$  \hspace{1cm} (7)

3. Make a graph of $k$ versus $L$ for each value of $L$ given in Table 1. Use this graph to determine an estimate for the constant, $E$, using a least squares best fit approach. Fit the data with an equation of the form

$$k = cL^b$$  \hspace{1cm} (8)

where $b$ is determined above (Note: students could use the Fit command in Mathematica or use the add trend line feature in MS Excel when plotting $k$ vs $L^b$). In fitting the data, you are determining the best value for the constant, $c$. Comparing equation 8 and your solution to (4) we see that $c$ is a multiple of $EI$. The constant $I$ can be calculated using equation 7 with the width and thickness (the dimensions for experimental data in Figure 1 are $w = 1.125$ and $t = 0.15$). From this calculation, determine $E$. 
Document your efforts and analysis in your project report. You can refer to sections of the text, credible websites, or other sources. Cite all sources, as appropriate. In particular, acknowledge any individuals who assist you. However, for each group, your work must be your own.

Submit your report with all graphs (clearly labeled) and supporting material (professionally presented). Use the notation provided in this discussion. Use the guidelines that you have been given for writing a science/engineering/mathematics report. You can use your calculus book to get examples of professional writing styles for mathematics and text (your instructors may have additional handouts or references).