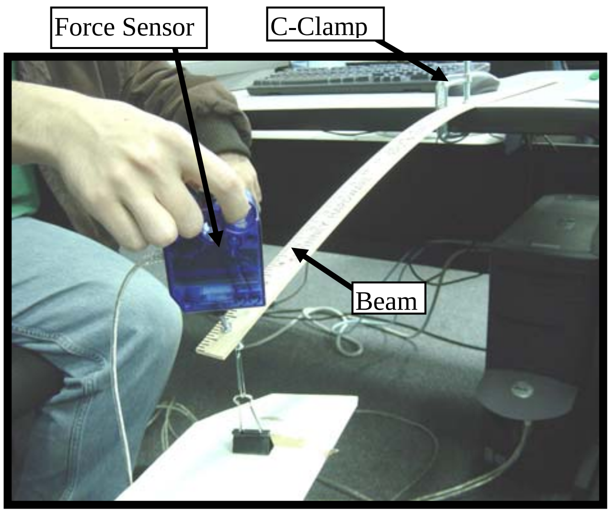
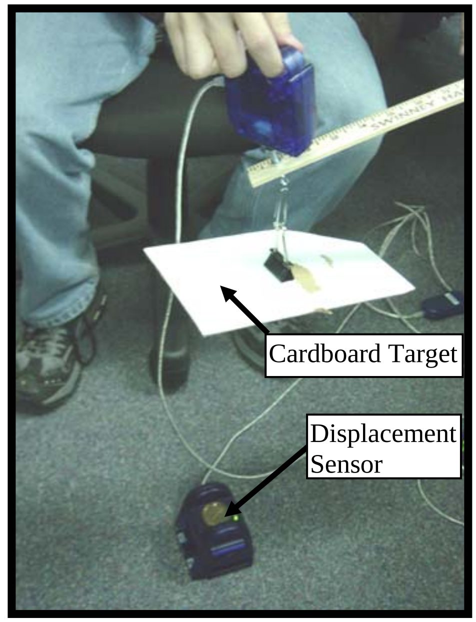
Mechanical Engineering-Mathematics

ILAP





Beam Deflection Using Real-time Sensors

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Interdisciplinary Lively Applications Project (ILAP)

Title: Beam Deflection Using Real-time Sensors

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Mathematics Classifications: Calculus II - MATH 2024
(Direct Integration of Differential Equations, Boundary Conditions)

Disciplinary Classification: Mechanics of Materials – ES 3023
(Beam Mechanics)

Prerequisite Skills:

1. Integration
2. Curve Fitting

Physical Concepts Examined:

1. Deflection of a cantilever beam

Materials Available:

1. Supplemental background material, Problem statement and discussion; Student
2. Sample solution; Instructor

Computing and Hardware Requirements:

DataStudio software, Yardstick, Clamp, Motion sensor/USB link, Force sensor/USB link, cardboard target, binder clip, paperclip, and super glue.

Class Requirements:

Half class period for background and instructions;
One class period for laboratory experiment

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INTRODUCTION:

Often in mechanical systems we can establish a general expression showing how an applied force tries to cause a change and meets with a resistance. The deflection of the end of a cantilever beam (one that sticks out from a wall with no support at the other end as shown in Figure 1) fits this same model:

When this ***system***	*…is acted upon by this* ***force***	*…it meets with this* ***resistance***	*…to change this* ***parameter***
Cantilever beam	Vertical force (load) (P)	Beam stiffness (k)	Tip deflection (δ)

The beam stiffness, *k*, summarizes a variety of factors that determine how the beam responds to a force. The stiffness is determined by the length from the fixed end to the point where the load is applied (*L*), a quantity called the moment of inertia of the cross sectional area of the beam (I), a material property of the beam called the modulus of elasticity (*E*), and a geometrical constant (*a*). The moment of inertia (*I*) is a function of the beam width (*w*) and the beam thickness (*t*). These parameters can be used for any beam loaded only at the end. The mathematical relationship is

$$P = k\,\delta \qquad (1)$$

where

$$k = a\,L^b\,EI \qquad (2)$$

so

$$P = \left(a\,L^b EI\right)\delta \qquad (3)$$

The variables *a* and *b* are constant real numbers that you will solve for later in the assignment.

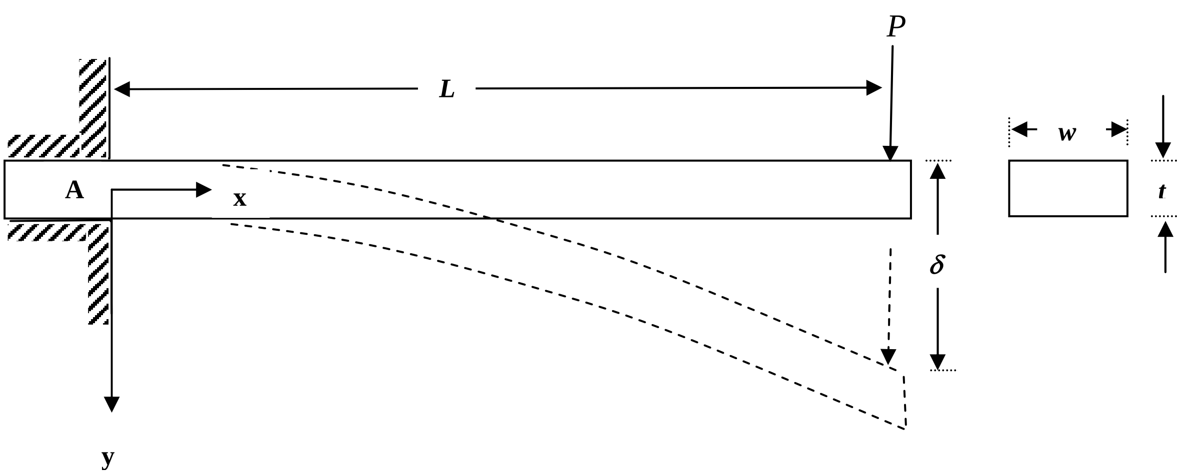


Figure 1. Diagram of cantilever beam

In this project you will experimentally determine the relationship between the beam stiffness, *k*, and beam length, *L*, by measuring the stiffness at different lengths using real time force and displacement sensors. You will then determine the theoretical solution for the beam stiffness by solving the governing differential equation and compare this theoretical result to your experimental results. Finally, you will use your experimental results along with the geometry of beam and the theoretical solution to estimate the modulus of elasticity, *E*, of the beam.

DATA COLLECTION PROCEDURE:

The first step is to experimentally determine the stiffness, *k*, of the beam for various lengths:

1. Hang the cardboard target from the end hook at the end of the beam (as shown on the cover illustration). The cardboard target should hang so that it is parallel to the floor.
2. Clamp the beam to a table with 24 inches of the beam extended out from the table surface. Ensure the beam is at a 90° angle with the table's edge.
3. The cardboard will be the target for a motion sensor. Place the motion sensor directly below the cardboard and attach the sensor to the USB link. Attach the USB link to the computer and launch DataStudio.
4. A force sensor, attached to a second USB link, will be used to push down on the unsupported end of the beam (see the front cover illustration). Attach the second USB link to the computer. At this point the DataStudio screen should look similar to Figure 2 below.

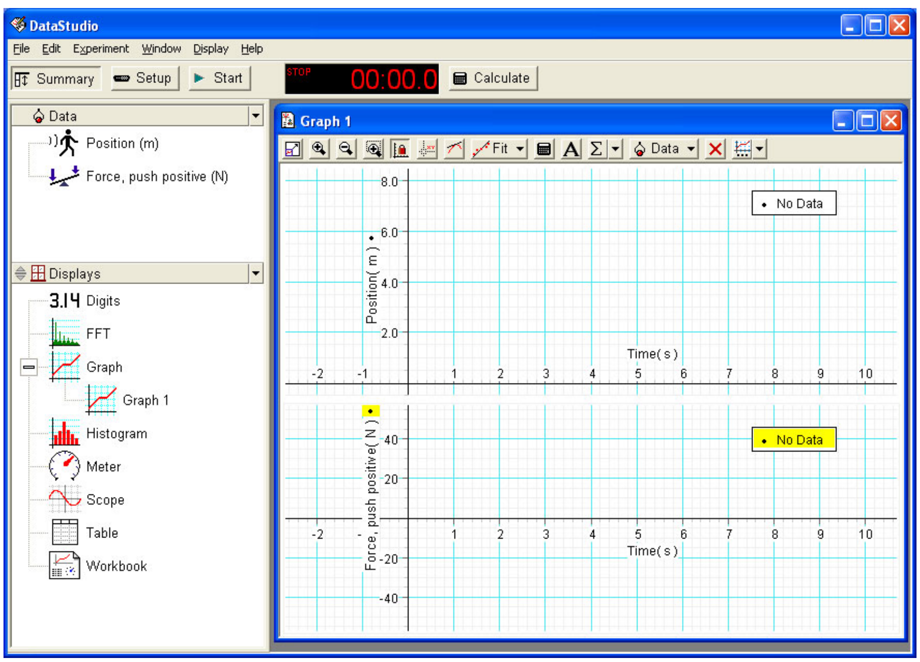


Figure 2. DataStudio software after connecting position and force sensors

5. Set up a Force vs. Position graph as follows:
   a. DataStudio will show two graphs: Force vs. Time (from the force sensor) and Position vs. Time (from the motion sensor).
   b. Drag "Position" from the "Data" area at the left ONTO the x-axis of the Force vs. Time graph.
   c. Close the Position vs. Time graph, which is not needed.
   d. You are ready to collect Force vs. Position data (the set-up should look like Figure 4)

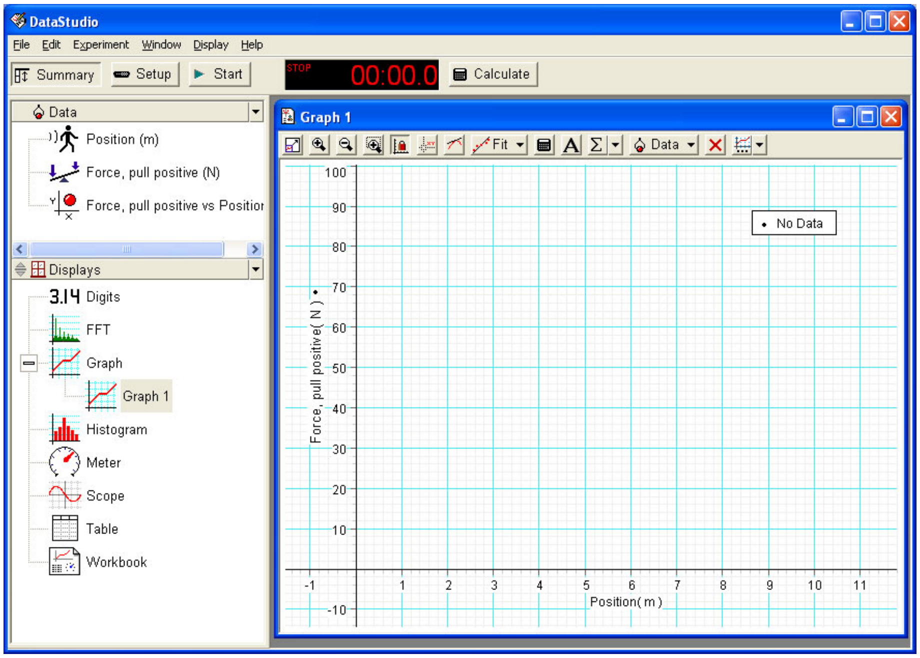


Figure 3. Force vs. Position graph after step 5

6. Collect force vs. position data (by pushing the start button) while you pull gently up on the force sensor with your hand and release it, as shown in Figure 4. You should get a relatively linear response, similar to that shown in Figure 5 (Note: the sign of the slope doesn't matter, it just depends on whether your sensor is setup for pushing or pulling as positive, this can be changed by clicking the set-up button) You may need to scale your graph using the autoscale icon (the top left icon in the graphing window). If your graph is not relatively linear, delete the data and repeat. Ensure that the displacement sensor is directly below the cardboard target and that the force load is perpendicular to the floor.

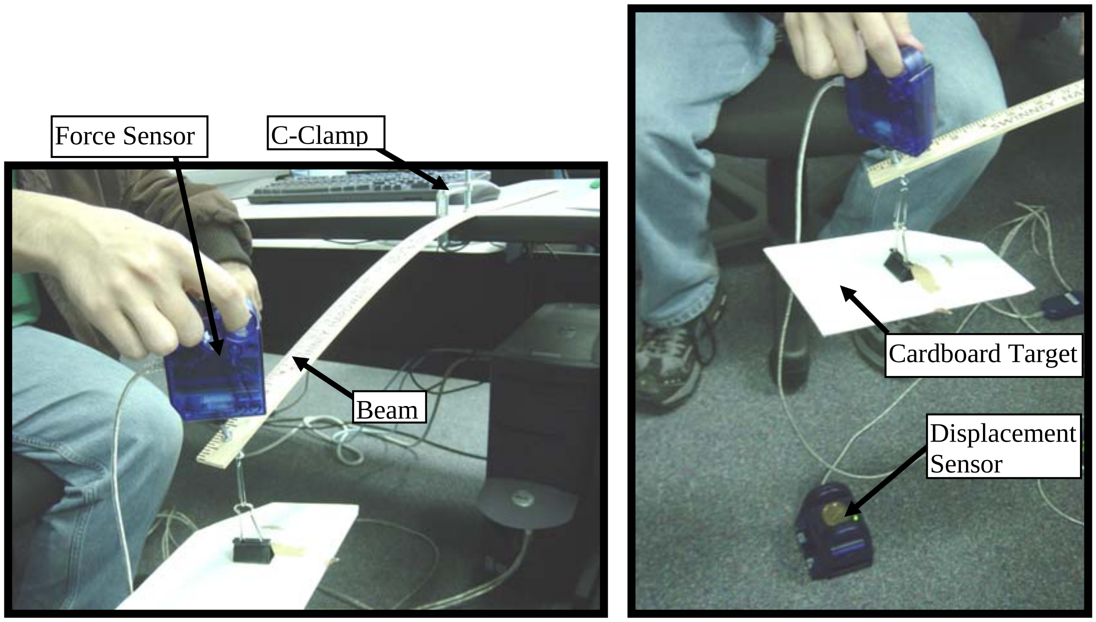


Figure 4. Experimental setup

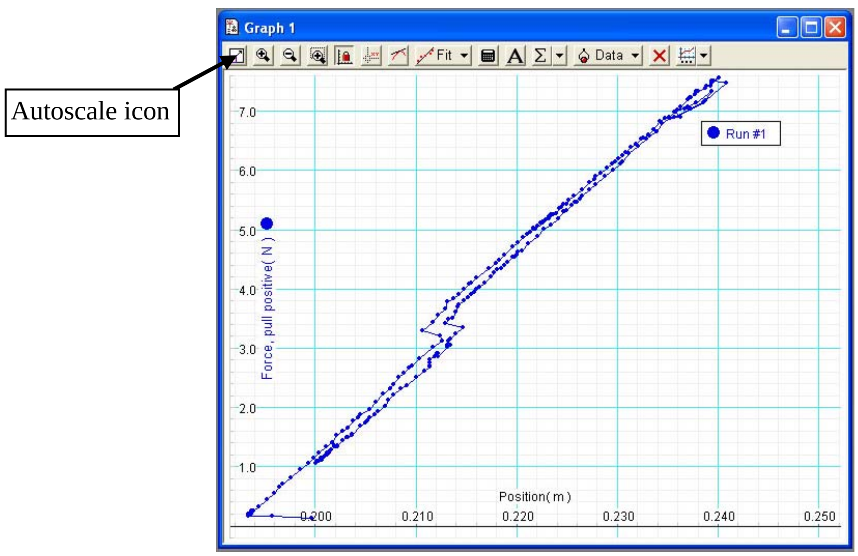


Figure 5. Typical Force vs Position graph

7. Select the data by drawing a box around it with the mouse and choose the linear fit under the Fit drop down list (Figure 6). The slope (m) displayed on the graph (see Figure 7) of the resulting linear fit is the beam stiffness, *k*, for this particular beam length, *L*. Record the value of the slope (beam stiffness) because you will be using this data later.

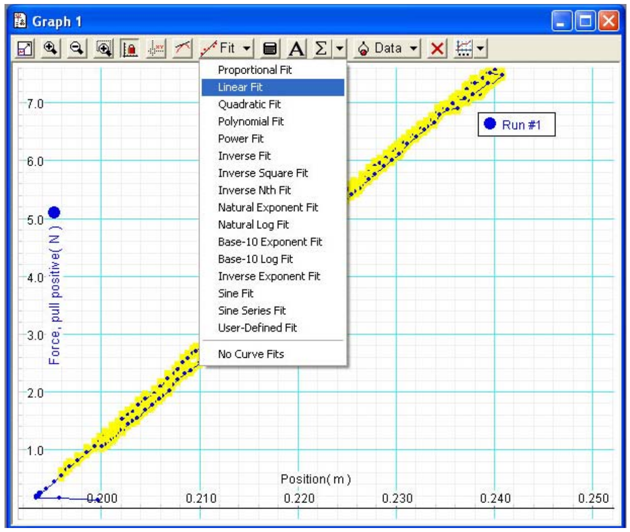


Figure 6. Select Linear Fit.

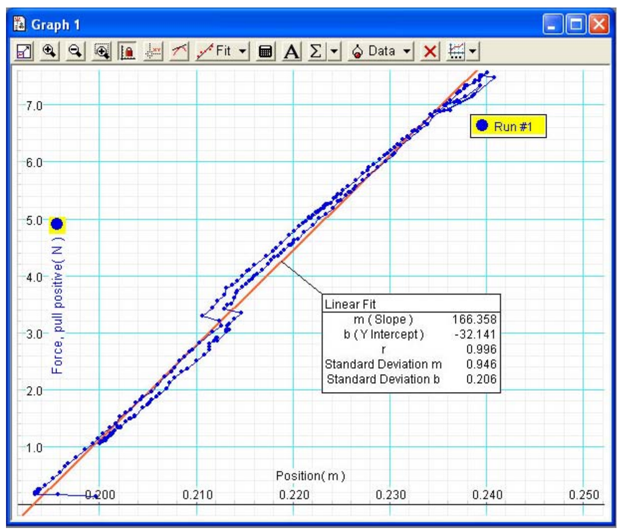


Figure 7. The slope of the linear fit is the beam stiffness.

8. Repeat steps 6 and 7 for beam lengths from 12 inches to 30 inches in 3 inch intervals, recording the slopes and beam lengths in a table of data. Save your activity file by selecting File, Save.

MATHEMATICAL ANALYSIS:

The governing differential equation for an end loaded cantilever beam is

$$\frac{d^2y}{dx^2} = \frac{P(L-x)}{EI} \tag{4}$$

where $y(x)$ is the vertical displacement evaluated at x (as shown in Figure 1). The deflection at the load point, δ, is the same as $y(x)\big|_{x=L}$. For a cantilever beam, the vertical displacement and the slope of the beam at the fixed end are both zero. Therefore the boundary conditions for this second-order differential equation are

$$y(0)=0$$
$$\left.\frac{dy}{dx}\right|_{x=0}=0 \qquad (5)$$

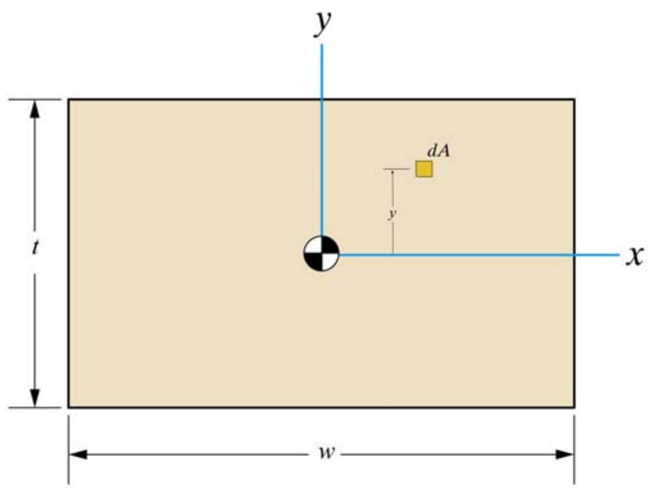
1. Solve equation (4) subject to these boundary conditions and use your result to determine the theoretical relationship between the beam stiffness and the parameters in equation 2. Recall from equation 1-3 that $P=k\,\delta=k\,y|_{x=L}=\left(a\;L^{b}EI\right)y|_{x=L}$. By solving differential equation 4, you are finding the unknown parameters, *a* and *b*.

2. The moment of inertia is sometimes called the second moment of area and is determined by the following equation:

$$I=\int_A y^2 dA \qquad (6)$$

   where *A* is the cross sectional area of the beam and *y* is measured from the x-axis that goes through the center of the beam. Show that for a rectangular beam with a width, *w*, and a thickness, *t*, the moment of inertia is

$$I=\int_A y^2 dA=\frac{1}{12}wt^3 \qquad (7)$$



3. Make a graph of measured beam stiffness, *k*, versus *L* for each of the beam lengths investigated. Use this graph to determine an estimate for the modulus of elasticity, *E*, using a least squares best fit approach. Fit the data with an equation of the form

$$k=cL^b \qquad (8)$$

   where *b* is determined above (Note: students could use the *Fit* command in Mathematica or use the *add trend line* feature in MS Excel when plotting *k* vs L^b). In fitting the data, you are determining the best value for the constant, *c*. Comparing equation 8 and equation 2 we see that *c* = *aEI*. The constant *a* is known from the solution to equation 4, and the moment of inertia, *I*, can be calculated using equation 7 with the beam width and thickness (the dimensions for your beam are *w* = 1.125 in and t = 0.15 in). When calculating the modulus of elasticity, *E*, remember to report the units, and make sure that all units in the calculations are consistent.

Document your efforts and analysis in your ILAP report. Explain any assumptions that have been made in the mathematical modeling of the problem which may account for discrepancies between what you have obtained **analytically** versus what you measured **experimentally**.

You can refer to sections of the text, credible websites, or other sources. Cite all sources, as appropriate. In particular, acknowledge any individuals who assist you. However, for each group, your work **must** be your own.

Submit your report with all graphs (clearly labeled) and supporting material (professionally presented). Use the notation provided in this discussion. Use the guidelines that you have been given for writing a science/engineering/mathematics report. (See "Technical Report Format and Writing Guide" at the TU ILAPs web site: http://www.ilaps.utulsa.edu/). You can use your calculus book to get examples of professional writing styles for mathematics and text (your instructors may have additional handouts or references).

References:

[1] "Using Real-time Sensors in the Classroom", Instructor Dr. Matt Ohland. Educational Research and Methods, ASEE Annual Conference and Exposition. June 20, 2004. Salt Lake City, Utah.